JMEPEG (2007) 16:220–223
DOI: 10.1007/s11665-007-9035-1

1059-9495/$19.00

Three-term Asymptotic Stress Field Expansion for Analysis of Surface Cracked Elbows in Nuclear Pressure Vessels

Fernando Labbe

(Submitted June 11, 2006)

Elbows with a shallow surface cracks in nuclear pressure pipes have been recognized as a major origin of potential catastrophic failures. Crack assessment is normally performed by using the *J-integral* approach. Although this one-parameter-based approach is useful to predict the ductile crack onset, it depends strongly on specimen geometry or constraint level. When a shallow crack exists (depth crack-to-thickness wall ratio less than 0.2) and/or a fully plastic condition develops around the crack, the J-integral alone does not describe completely the crack-tip stress field. In this paper, we report on the use of a three-term asymptotic expansion, referred to as the J–A_2 methodology, for modeling the elastic-plastic stress field around a three-dimensional shallow surface crack in an elbow subjected to internal pressure and out-of-plane bending. The material, an A 516 Gr. 70 steel, used in the nuclear industry, was modeled with a Ramberg–Osgood power law and flow theory of plasticity. A finite deformation theory was included to account for the highly nonlinear behavior around the crack tip. Numerical finite element results were used to calculate a second fracture parameter A_2 for the J–A_2 methodology. We found that the used three-term asymptotic expansion accurately describes the stress field around the considered three-dimensional shallow surface crack.

Keywords asymptotic stress fields, constraint, nonlinear fracture mechanics, pressure vessels

1. Introduction

Flaws in power plant pipes under normal operating conditions typically initiate on the inner surface of the elbow walls. They grow by fatigue and/or stress corrosion forming a surface crack. As a consequence, shallow surface cracks (depth crack-to-thickness wall ratio less than 0.2) are often encountered and hence have been recognized as a major origin of potential catastrophic failure. In order to prevent these failures, fracture mechanics methodologies are normally applied to evaluate the structural integrity of pressure pipes.

The application of conventional fracture mechanics techniques relies on the notion that a single parameter, such as *J-integral*, characterizes the resistance of the material to fracture (Ref 1, 2). The *J-integral* is used to measure the intensity of the stress and deformation fields on the singular one-term crack-tip solution for a nonlinear material. It is referred to as the HRR singularity field since Hutchinson and Rice (Ref 3) and Rosengren (Ref 4). However, the applicability of this one-parameter-based approach is limited to high constraint crack geometry such as deep crack in bending loading. When shallow surface crack exists and/or extended plastic conditions around the crack are meet, J-integral parameter alone does not describe completely the field around the crack tip.

The limited ability of a single parameter J to fully characterize crack-tip conditions for certain geometry and load conditions is still an open problem. To solve it, an approach recently proposed is establishing a multi-term asymptotic crack-tip stress field *expansion* (Ref 5).

A two-term asymptotic stress field expansion in fracture mechanics has been carried out by O'Dowd et al. (Ref 6-8) as well as Betegon and Hancock (Ref 9). In the two-parameter approach, denoted as J–Q methodology, a second parameter Q measures the degree of triaxiality and constraint of the stress field. Applications of J–Q methodology to cracked cylindrical specimen has been performed by Donoso (Ref 10) and, in the case of flawed pressure vessels, by Labbe (Ref 11). Both works have shown that J–Q methodology can only render high-quality predictions for deep cracks.

The three-term asymptotic stress field expansion (Ref 12-15), referred to as the J–A_2 methodology, is controlled by two amplitude parameters that describe the stress field in the vicinity of the crack-tip. This paper presents an application of the J–A_2 methodology to analyze the stress field in the area close to a three-dimensional shallow crack in a pressurized pipe. The considered shallow surface crack was represented as a semi-elliptic inner surface flaw. In order to solve with high accuracy the stresses, a high-density mesh with three-dimensional higher order finite elements was located around the crack-tip. Large nonlinear strain effects around the crack-tip were also included on the finite element model and we used stress values in the region close to the crack-tip to evaluate a second parameter A_2 for the J–A_2 methodology. We found that the used three-term asymptotic expansion accurately describes the stress field in the area which is significant for fracture.

Fernando Labbe, Mechanical Engineering Department, Universidad Tecnica Federico Santa Maria, Casilla 110-V, Valparaiso, Chile. Contact e-mail: fernando.labbe@usm.cl

2. Theoretical Background

2.1 Two-Parameter Fracture Mechanics: J-Q Methodology

According to Yang et al. (Ref 12) the two-term asymptotic expansion that represents close to the crack tip, but still outside of the zone of finite strain, the actual stress field is:

$$\sigma_{ij} = \sigma_0 \left[\frac{J}{\alpha\sigma_0\varepsilon_0 I_n r}\right]^{1/n+1} \phi_{ij}(\theta, n) + Q\sigma_0 \left[\frac{r}{J/\sigma_0}\right]^{q} \psi_{ij}(\theta, n), \quad \text{(Eq 1)}$$

where r and θ are the local cylindrical coordinates centered at the crack tip. The normalizing factor I_n and the angular distribution of the dimensionless constants ϕ_{ij} (θ, n) and ψ_{ij} (θ, n) are functions of the strain-hardening exponent n. The first term in this expansion corresponds to the HRR field (Ref 3, 4), with the J-integral parameter as amplitude. The dimensionless parameter Q measures the amplitude of the second term.

Numerical results of O'Dowd et al. (Ref 6-8) show that the second term is approximately independent of the radial distance, $|q| << 1$, and the corresponding $\psi_{ij}(\theta, n)$ functions do not depend on θ. This means that the second term in the expansion (1) acts as a hydrostatic uniform stress. Therefore, relation (1) may be written as:

$$\sigma_{ij} = (\sigma_{ij})_{\text{HRR}} + Q \cdot \sigma_0 \cdot \delta_{ij}, \quad \text{(Eq 2)}$$

where $(\sigma_{ij})_{\text{HRR}}$ is the HRR field and Q represents a triaxiality parameter. A negative Q means that the hydrostatic stress is reduced (low stress triaxiality) in comparison with the reference state (HRR field), and therefore the J-integral approach is no longer valid. On the other hand, structures with $Q \geq 0$ exhibit high stress triaxiality, good agreement with the HRR fields, and they are consequently highly constrained.

From Eq (2), we can evaluate Q as the difference between the actual full-stress field, and the HRR field:

$$Q = \frac{\sigma_{\theta\theta} - (\sigma_{\theta\theta})_{\text{HRR}}}{\sigma_0}, \quad \text{(Eq 3)}$$

for $\theta = 0$ and normalized distance $r = 2J/\sigma_0$.

Unfortunately, this approach represents well the stress only for deep cracks.

2.2 Three-Term Asymptotic Expansion: J–A_2 Methodology

The general elastic-plastic behavior of hardening material is described by the Ramberg–Osgood power-law relation. Generalizing this equation to a multidimensional stress by the J_2 deformation theory and using small deformation theory (Ref 12-15), we can obtain the three-term asymptotic crack-tip stress field which is controlled by only two amplitude parameters, the J-integral parameter and the A_2 parameter:

$$\frac{\sigma_{ij}}{\sigma_0} = \bar{A}_1 \left[\left(\frac{r}{L}\right)^{-\frac{1}{(n+1)}} \sigma_{ij}^{(1)}(\theta, n) + A_2 \left(\frac{r}{L}\right)^{s_2} \sigma_{ij}^{(2)}(\theta, n) + A_2^2 \left(\frac{r}{L}\right)^{2s_2+\frac{1}{(n+1)}} \sigma_{ij}^{(3)}(\theta, n)\right], \quad \text{(Eq 4)}$$

where A_1 is given by

$$\bar{A}_1 = \left[\frac{J}{\alpha\sigma_0\varepsilon_0 I_n L}\right]^{1/(n+1)}. \quad \text{(Eq 5)}$$

In Eq (4), r and θ are the local cylindrical coordinates centered at the crack tip, and $\sigma_{ij}{}^{(1)}(\theta, n)$, $\sigma_{ij}{}^{(2)}(\theta, n)$ y $\sigma_{ij}{}^{(3)}(\theta, n)$ are the angular distribution of the dimensionless constants. These are functions of the strain-hardening exponent n, the stress power exponent s_2 and the factor I_n. The latter depends only on the hardening exponent n. L is a characteristic length parameter.

A critical part in the evaluation of the parameters J and Q, is the evaluation of the crack-tip stress field around the defect. Defects such as surface cracks produce complex stress fields within the area close to the crack-tip due to three-dimensional effects, large deformations and microstructural fracture process. Therefore, the evaluation of these parameters requires a detailed nonlinear three-dimensional stress analysis.

The first term in the expansion (Eq 4) corresponds to the HRR field with the J-integral parameter as amplitude. The dimensionless parameter A_2 is an undetermined parameter that can be related to the loading and geometry configuration. From Eq (4), the A_2 parameter can be calculated by applying a least square fitting to the values of the stress in the area close to the crack-tip.

A negative value of A_2 means that the hydrostatic stress is reduced (low stress triaxiality) in comparison with the reference state (HRR field), and therefore the J-integral approach is no longer valid. On the other hand, structures with $A_2 \geq 0$ exhibit high stress triaxiality, good agreement with the HRR fields, and consequently the J-integral approach is valid.

3. Example

3.1 Numerical Calculations

As an example, consider an elbows with a shallow surface cracks in nuclear pressure pipes. A typical material in the nuclear industry; a high strength steel A 516 Gr. 70 steel. In elastic-plastic fracture-mechanics, the true stress strain is usually approximated by the Ramberg–Osgood power-law relation:

We took, in the Ramberg–Osgood power-law relation, the reference stress σ_0 as equal to the 0.2% offset yield stress and the reference strain ε_0 as equal to σ_0/E, being E the Young's modulus. α is a hardening parameter and n is the strain-hardening exponent. The values used in this model were σ_0 = 360 MPa. α = 1.07 and n = 13.

The material was modeled with incremental theory of plasticity, Von Mises criterion, and isotropic hardening rule. In the finite element model (FEM), we also considered finite strain plasticity and large strain/rotation effects around the crack tip.

The finite element code ABAQUS (Ref 16) was used in the analysis of the elbow using a total number of 11,200 high-order 20-nodes, isoparametric elements. A detailed meshing of the semi-elliptic surface crack is presented in Fig. 1.

We considered an inner radius R_i = 152 mm (6 in.) and a wall thickness t = 10 mm for the pressurized elbow with a shallow surface crack. The crack was modeled as a three-dimensional semi-elliptic inner surface flaw with a depth-to-length ratio of $a/2c$ = 0.1. The assumed shallow crack had a depth-to-thickness wall ratio of a/t = 0.125. The finite element model of the half-length pipe was sufficiently long (2,000 mm) to avoid end effects on loading. Due to load and geometry symmetry, by using appropriate boundary conditions imposed on the planes of symmetry, it was necessary to consider only one-fourth of the complete elbow-pipe.

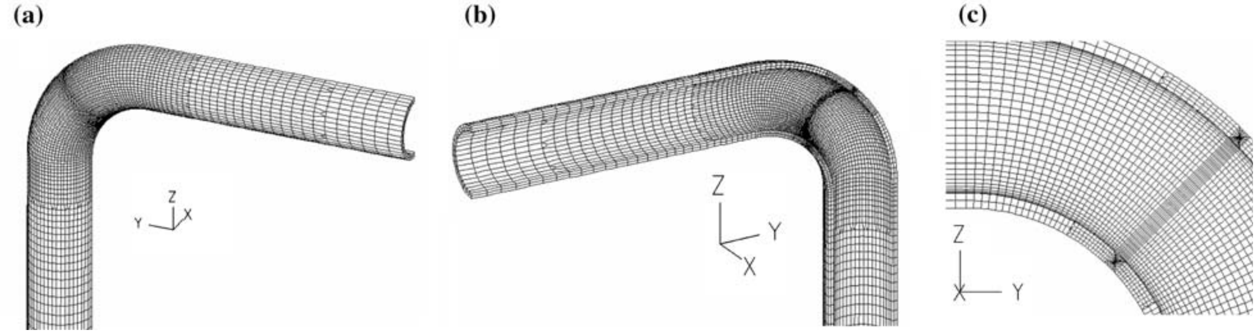

Fig. 1 (a) Outer view of the elbow finite element model, inner radius R_i = 152.4 mm (6″), thickness t = 10 mm, (b) Inner view of the elbow finite element model, (c) Finite element crack detailed

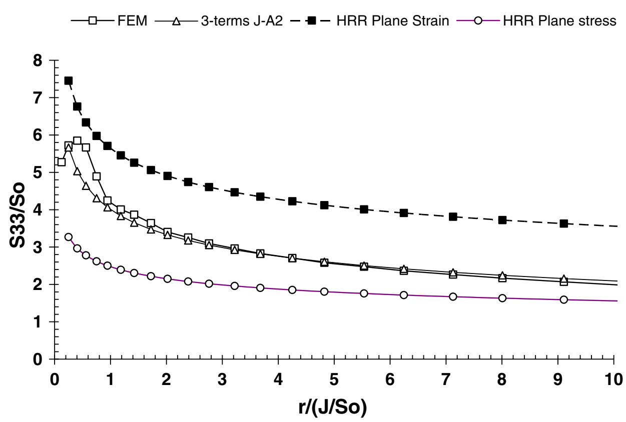

Fig. 2 Normalized FEM, J-A_2, HRR plane stress and strain versus normalized crack-tip distance; applied pressure loading p = 18 MPa and out-of-Plane load of 300 N-m

The applied pressure loading in the pipe was 18 MPa and we considered an out-of-plane bending of 300 N-m. We evaluated the numerical stresses at the critical position, which occurred on the minor ellipse semi-axis of the crack.

The J-integral values were calculated by a virtual crack extension method implemented by using ABAQUS (Ref 16).

3.2 Numerical Results

The A_2 parameter for the three-term asymptotic solution was obtained from Eq (4) by fitting the whole-field stress values computed by finite element around the crack-tip. The value calculated by a least square adjustment was $A_2 = -2.5442$. The negative value of A_2 allows us to conclude that the hydrostatic stress is reduced (low stress triaxiality) in comparison with the reference state (HRR field). This means that the J-integral approach should not be appropriate for the assessment of the considered crack.

Figure 2 shows a finite element method (FEM) results, the HRR singularity stress field and a three-term approximation (J–A_2 field) for the normalized opening stress around the shallow crack, versus the crack-tip distance normalized by using J/σ_0. The normalizing factor J/σ_0 for the crack-tip distance is useful to set the local size scale on which large strains and stresses develop and the processes of microscopic ductile fracture occur.

It can be observed in Fig. 2 that, as expected, the HRR-field is far from the finite element solutions. From these results, we concluded that the J-approach based only on the HRR asymptotic one-term solution cannot characterize three-dimensional shallow cracks in elbows subjected to internal pressure and out-of-plane bending.

On the other hand, the data plotted in Fig. 2 allowed us to verify the quality of the near-tip stress fields rendered by the three-term elastic-plastic asymptotic expansion. As shown in Fig. 2, the J–A_2 three-term asymptotic solution is essentially similar to the finite element solutions over the interval $1 < r/(J/\sigma_0) < 10$; J–A_2 fields match very well with the FEM results. This allows us to conclude that, for failure assessment of a shallow surface cracks in pressure pipes, it is convenient to use the Fracture Mechanics approach based on three-term asymptotic expansion, with parameters J and A_2.

Moreover, as observed in Fig. 2, for the very near crack-tip region, $r < 1(J/\sigma_0)$, there are a drastic breakdown in the opening stress behavior. This phenomenon is consistent with the existence of large nonlinear effects on that region that invalidate the HHR asymptotic stress field based on the small strain theory.

4. Summary and Conclusions

The conventional fracture mechanics techniques use the *J-integral* as a measure of the intensity of the stress and deformation fields on the singular one-term crack-tip solution for nonlinear materials. This approach, based on a single parameter, yields the referred to as HHR asymptotic stress field. However, its application is limited to deep cracks in bending loading. The J-integral parameter alone does not describe completely the field around a shallow surface crack with or without surrounding plastic conditions. Although the two-term asymptotic stress field expansion, denoted as J–Q methodology, allows also including in the analysis the degree of triaxiality and constraint of the stress field, this methodology can only render high-quality predictions for deep cracks.

Shallow surface cracks (depth crack-to-thickness wall ratio less than 0.2) in power plant components initiate at the inner surface of elbows in pressurized pipes. As pointed out above, for this geometry and load conditions, the J-integral and the J–Q methodology have limited ability to characterize the crack-tip behavior. Therefore, in order to evaluate the whole-field stress for a shallow surface crack in a pressure pipe, we applied the three-term asymptotic crack-tip stress field expansion, referred to as the J–A_2 methodology.

The applied $J–A_2$ methodology is controlled by two amplitude parameters that allowed us to describe the stress field in the vicinity of the crack-tip. We represented the shallow surface crack as a semi-elliptic inner surface flaw. In order to obtain high accuracy in the stresses within the area close to the crack-tip, a high-density mesh with three-dimensional higher order finite elements was located around the crack-tip. Large nonlinear strain effects around the crack-tip were also included on the finite element model and we evaluated a second parameter A_2 for the $J–A_2$ methodology by fitting the whole-field stress values computed by finite element around the crack-tip.

We found that the three-term elastic-plastic asymptotic expansion accurately describes the stress field in the region close to the crack-tip which is significant for fracture. In this region, the $J–A_2$ three-term asymptotic solution and the finite element solutions coincided. For the very near crack-tip area, we observed a drastic breakdown in the opening stress behavior that should be linked with the existence of large nonlinear effects in that region.

We argue that the nonlinear effects, observed in the region close to the crack-tip, invalidate the HHR asymptotic stress field based on the small strain theory. This means that the J-approach, based only on the HRR asymptotic one-term solution, cannot be used to analyze the stress field in the area close to a three-dimensional shallow crack in a pressurized pipe.

We conclude that the failure assessment, of three-dimensional shallow cracks in elbows subjected to internal pressure and out-of-plane bending, requires an approach based on three-term asymptotic expansion with parameters, J and A_2.

Although in the example shown above, we applied a three-parameter fracture mechanical approach to describe the stress field ahead of the crack-tip, more analysis should be performed to validate the proposed approach. Specifically, in the short term, we are planning to examine: a range of pressure loading; results for tension/bend loading; small strain vs. large strain; use of SSY reference field vs. use of the HRR (see (Ref 17)); a range of different materials; the mesh sensitivity of the result.

Acknowledgement

The author acknowledges the financial support of DGIP-CHILE, Research Project No. 25.06.22 from the Universidad Técnica Federico Santa Maria.

References

1. J.W. Hutchinson, Fundamentals of the Phenomenological Theory of Nonlinear Fracture Mechanics, *J. Appl. Mech.*, 1983, **50**, p 1042–1051
2. J.D. Landes, "Elastic-plastic fracture mechanics: where has it been? where is it going," *Fatigue and Fracture Mechanics, 30th Volume*, P.C. Paris and K.L. Jerina, Eds., ASTM STP 1360, 1999, p 3–18
3. J.W. Hutchinson, Singular Behavior at the end of a Tensile Crack in a Hardening Material, *J. Mech. Phys. Solids*, 1968, **16**, p 13–31
4. J.R. Rice and G.F. Rosengren, Plane Strain Deformation near a crack tip in a Power Law Material, *J. Mech. Phys. Solids*, 1968, **16**, p 1–12
5. S.M. Sharma and N. Aravas, Determination of Higher-order terms in Asymptotic Elastoplastic Crack Tip Solutions, *J. Mech. Phys. Solids*, 1991, **39**, p 1043–1072
6. N.P. O'Dowd and C.F. Shih, Family of Crack-tip Fields Characterized by a Triaxiality parameter-I Structure of Fields, *J. Mech. Phys. Solids*, 1991, **39**, p 898–1015
7. N.P. O'Dowd and C.F. Shih, Family of Crack-tip Fields Characterized by a Triaxiality parameter-II Fracture Applications, *J. Mech. Phys. Solids*, 1992, **40**, p 939–963
8. N.P. O'Dowd and C.F. Shih, "Two-Parameter Fracture Mechanics: Theory and Applications," Fracture Mechanics: Twenty-Fourth Volume, ASTM STP 1207, American Society for Testing and Materials, Philadelphia, 1994, p 21–47
9. C. Betegon and J.W. Hancock, Two-parameter Characterization of Elastic-Plastic Crack-tip Fields, *J. Appl. Mech.*, 1991, **68**, p 104–110
10. J.R. Donoso and F. y Labbé, Q Stresses and Constraint Behavior of the Notched Cylindrical Tensile Specimen, *Eng. Frac. Mech.*, 2001, **68**, p 487–496
11. F. Labbé, and J.R. Donoso, "Nonlinear Finite Element Modeling of Surface Cracks in a Nuclear Pressure Pipe," *Proceedings of the First MIT Conference on Computational Fluid and Solids Mechanics* Cambridge, MA, Massachussetts Institute of Technology, J.K. Bathe, Ed., June 2001
12. S. Yang, Y.J. Chao, and M.A. Sutton, Higher Order Asymptotic Crack Fields in a Power-law Hardening Material, *Eng. Frac. Mech.*, 1993, **45**, p 1–20
13. Y.J Chao, S. Yang, and M.A. Sutton, On the Fracture of Solids Characterized by One or Two Parameters: Theory and Practice, *J. Mech. Phys. Solids*, 1994, **42**, p 629–647
14. Y.J. Chao and X.K. Zhu, "J-A_2 Characterization of Crack-Tip Fields: Extent of J-A_2 Dominance and Size Requirements," *Int. J. Frac.*, 1998, **89**, p 285–307
15. Y.J. Chao, X.K. Zhu, P.S. Lam, M.R. Louthen, and N.C. Iyer, "Application of the Two-Parameter $J–A_2$ Description to Ductile Crack Growth," *Fatigue and Fracture Mechanics: 31th Volume*, G.R. Halford and J.P. Gallagher, Eds., American Society for Testing and Materials, ASTM STP 1389, 2000, p 165–182
16. ABAQUS Theory manual, version 5.8, 1999, Hibbitt, Karlsson and Sorensen, Inc., Providence, RI
17. N.P. O'Dowd, Applications of Two Parameter Approaches in Elastic-Plastic Fracture Mechanics *Eng. Frac. Mech*., 1995, **52**, p 445–465